¿Ya se derivar?

II CASTING DERIVADAS 2010-11

Derivar y simplificar las siguientes funciones:

1

$$y = 18 - 5x^3 + 4\sqrt{x}$$

2

$$y = \sin^3(x)\cos^3(x)$$

3

$$y = \frac{\tan(x)}{x\sin(x)}$$

4

$$y = x\,e^{-\frac{1}{x^2}}$$

5

$$y = 5\log\left(\sin\left(5x^5\right)\right) - 5$$

6

$$y = \frac{3}{\sqrt[7]{x^4}} - \arccos x$$

7

$$y = x^4 \sin(x)$$

8

$$y = \frac{1-\cos 2x}{1+\cos 2x}$$

9

$$y = (x - \sin(x))^{\sqrt{x}}$$

10

$$y = \frac{x}{2}\sqrt{x^2-64} - 32\log\left(x + \sqrt{x^2-64}\right)$$

tan x es tangente de x
log x es logaritmo neperiano de x : ln x

Soluciones

1

WolframAlpha computational knowledge engine

derive 18-5x^3+4 sqrt(x)

Derivative:

Hide steps

$$\frac{d}{dx}\left(18-5x^3+4\sqrt{x}\right)=\frac{2}{\sqrt{x}}-15x^2$$

Possible derivation:

$$\frac{d}{dx}\left(-5x^3+4\sqrt{x}+18\right)$$

Differentiate the sum term by term and factor out constants:

$$=-5\left(\frac{d}{dx}(x^3)\right)+4\left(\frac{d}{dx}(\sqrt{x})\right)+\frac{d}{dx}(18)$$

The derivative of 18 is zero:

$$=-5\left(\frac{d}{dx}(x^3)\right)+4\left(\frac{d}{dx}(\sqrt{x})\right)+0$$

The derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$:

$$=4\,\frac{1}{2\sqrt{x}}-5\left(\frac{d}{dx}(x^3)\right)$$

The derivative of x^3 is $3x^2$:

$$=\frac{2}{\sqrt{x}}-5\left(3x^2\right)$$

2

derive sin^3x cos^3x

Derivative:

Hide steps

$$\frac{d}{dx}\left(\sin^3(x)\cos^3(x)\right) = \frac{3}{8}\sin(2x)\sin(4x)$$

Possible derivation:

$$\frac{d}{dx}\left(\sin^3(x)\cos^3(x)\right)$$

Use the product rule, $\frac{d}{dx}(u\,v) = v\frac{du}{dx} + u\frac{dv}{dx}$, where $u = \cos^3(x)$ and $v = \sin^3(x)$:

$$= \cos^3(x)\left(\frac{d}{dx}\left(\sin^3(x)\right)\right) + \sin^3(x)\left(\frac{d}{dx}\left(\cos^3(x)\right)\right)$$

Use the chain rule, $\frac{d}{dx}\left(\sin^3(x)\right) = \frac{du^3}{du}\frac{du}{dx}$, where $u = \sin(x)$ and $\frac{du^3}{du} = 3u^2$:

$$= \sin^3(x)\left(\frac{d}{dx}\left(\cos^3(x)\right)\right) + \cos^3(x)\left(3\sin^2(x)\left(\frac{d}{dx}(\sin(x))\right)\right)$$

The derivative of $\sin(x)$ is $\cos(x)$:

$$= \sin^3(x)\left(\frac{d}{dx}\left(\cos^3(x)\right)\right) + 3\sin^2(x)\cos(x)\cos^3(x)$$

Use the chain rule, $\frac{d}{dx}\left(\cos^3(x)\right) = \frac{du^3}{du}\frac{du}{dx}$, where $u = \cos(x)$ and $\frac{du^3}{du} = 3u^2$:

$$= \sin^3(x)\left(3\cos^2(x)\left(\frac{d}{dx}(\cos(x))\right)\right) + 3\sin^2(x)\cos^4(x)$$

The derivative of $\cos(x)$ is $-\sin(x)$:

$$= 3\sin^2(x)\cos^4(x) + 3(-\sin(x))\sin^3(x)\cos^2(x)$$

3

derive tanx/(x senx)

Derivative:

Hide steps

$$\frac{d}{dx}\left(\frac{\tan(x)}{x\sin(x)}\right) = \frac{(x\tan(x) - 1)\sec(x)}{x^2}$$

Possible derivation:

$$\frac{d}{dx}\left(\frac{\sec(x)}{x}\right)$$

Use the quotient rule, $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,\frac{du}{dx} - u\,\frac{dv}{dx}}{v^2}$, where $u = \sec(x)$ and $v = x$:

$$= \frac{x\left(\frac{d}{dx}(\sec(x))\right) - \sec(x)\left(\frac{d}{dx}(x)\right)}{x^2}$$

The derivative of $\sec(x)$ is $\tan(x)\sec(x)$:

$$= \frac{x(\tan(x)\sec(x)) - \sec(x)\left(\frac{d}{dx}(x)\right)}{x^2}$$

The derivative of x is 1:

$$= \frac{x\tan(x)\sec(x) - 1\sec(x)}{x^2}$$

$\sec(x)$ is the secant function »

4

derive x e^(-1/x^2)

Derivative:

Hide steps

$$\frac{d}{dx}\left(\frac{x}{e^{\frac{1}{x^2}}}\right) = \frac{e^{-\frac{1}{x^2}}\left(x^2+2\right)}{x^2}$$

Possible derivation:

$$\frac{d}{dx}\left(e^{-\frac{1}{x^2}}\,x\right)$$

Use the product rule, $\frac{d}{dx}(u\,v) = v\,\frac{du}{dx} + u\,\frac{dv}{dx}$, where $u = e^{-\frac{1}{x^2}}$ and $v = x$:

$$= e^{-\frac{1}{x^2}}\left(\frac{d}{dx}(x)\right) + x\left(\frac{d}{dx}\left(e^{-\frac{1}{x^2}}\right)\right)$$

Use the chain rule, $\frac{d}{dx}\left(e^{-\frac{1}{x^2}}\right) = \frac{de^u}{du}\,\frac{du}{dx}$, where $u = -\frac{1}{x^2}$ and $\frac{de^u}{du} = e^u$:

$$= x\left(e^{-\frac{1}{x^2}}\left(\frac{d}{dx}\left(-\frac{1}{x^2}\right)\right)\right) + e^{-\frac{1}{x^2}}\left(\frac{d}{dx}(x)\right)$$

The derivative of x is 1:

$$= e^{-\frac{1}{x^2}}\,x\left(\frac{d}{dx}\left(-\frac{1}{x^2}\right)\right) + 1\,e^{-\frac{1}{x^2}}$$

Factor out constants:

$$= e^{-\frac{1}{x^2}}\,x\left(-\left(\frac{d}{dx}\left(\frac{1}{x^2}\right)\right)\right) + e^{-\frac{1}{x^2}}$$

The derivative of $\frac{1}{x^2}$ is $-\frac{2}{x^3}$:

$$= e^{-\frac{1}{x^2}} - e^{-\frac{1}{x^2}}\,x\left(-\frac{2}{x^3}\right)$$

5

derive 5ln(sin(5x^5))-5

Derivative:

Hide steps

$$\frac{d}{dx}(5\log(\sin(5x^5))-5) = 125x^4\cot(5x^5)$$

Possible derivation:

$$\frac{d}{dx}(5\log(\sin(5x^5))-5)$$

Differentiate the sum term by term and factor out constants:

$$= 5\left(\frac{d}{dx}(\log(\sin(5x^5)))\right) + \frac{d}{dx}(-5)$$

The derivative of -5 is zero:

$$= 5\left(\frac{d}{dx}(\log(\sin(5x^5)))\right) + 0$$

Use the chain rule, $\frac{d}{dx}(\log(\sin(5x^5))) = \frac{d\log(u)}{du}\frac{du}{dx}$, where $u = \sin(5x^5)$ and $\frac{d\log(u)}{du} = \frac{1}{u}$:

$$= 5\left(\csc(5x^5)\left(\frac{d}{dx}(\sin(5x^5))\right)\right)$$

Use the chain rule, $\frac{d}{dx}(\sin(5x^5)) = \frac{d\sin(u)}{du}\frac{du}{dx}$, where $u = 5x^5$ and $\frac{d\sin(u)}{du} = \cos(u)$:

$$= 5\csc(5x^5)\left(\cos(5x^5)\left(\frac{d}{dx}(5x^5)\right)\right)$$

Factor out constants:

$$= 5\cot(5x^5)\left(5\left(\frac{d}{dx}(x^5)\right)\right)$$

The derivative of x^5 is $5x^4$:

$$= 25(5x^4)\cot(5x^5)$$

6

derive 3/x^(4/7)-arc cos x

Derivative:

Show steps

$$\frac{d}{dx}\left(\frac{3}{x^{4/7}} - \cos^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}} - \frac{12}{7\,x^{11/7}}$$

$\cos^{-1}(x)$ is the inverse cosine function »

7

derivative of x^4 sin x

Derivative:

Hide steps

$$\frac{d}{dx}\left(x^4 \sin(x)\right) = x^3\,(4\sin(x) + x\cos(x))$$

Possible derivation:

$$\frac{d}{dx}\left(x^4 \sin(x)\right)$$

Use the product rule, $\frac{d}{dx}(u\,v) = v\,\frac{du}{dx} + u\,\frac{dv}{dx}$, where $u = x^4$ and $v = \sin(x)$:

$$= x^4\left(\frac{d}{dx}(\sin(x))\right) + \sin(x)\left(\frac{d}{dx}(x^4)\right)$$

The derivative of $\sin(x)$ is $\cos(x)$:

$$= \sin(x)\left(\frac{d}{dx}(x^4)\right) + x^4\cos(x)$$

The derivative of x^4 is $4\,x^3$:

$$= x^4\cos(x) + (4\,x^3)\sin(x)$$

8

derive (1-cos2x)/(1+cos2x)

Derivative: Hide steps

$$\frac{d}{dx}\left(\frac{1-\cos(2x)}{1+\cos(2x)}\right) = 2\tan(x)\sec^2(x)$$

9

derive (x-sinx)^sqrt(x)

Derivative: Hide steps

$$\frac{d}{dx}\left((x-\sin(x))^{\sqrt{x}}\right) = \frac{(x-\sin(x))^{\sqrt{x}}\left(\log(x-\sin(x)) - \frac{2x(\cos(x)-1)}{x-\sin(x)}\right)}{2\sqrt{x}}$$

10

derive x/2 sqrt(x^2-64)-32 ln(x+sqrt(x^2-64))

Derivative: Show steps

$$\frac{d}{dx}\left(\frac{1}{2}x\sqrt{x^2-64} - 32\log\left(x+\sqrt{x^2-64}\right)\right) = \sqrt{x^2-64}$$

$\log(x)$ is the natural logarithm »